

One-step deterministic polarization entanglement purification using spatial entanglement*

Yu-Bo Sheng^{1,2} and Fu-Guo Deng^{1†}

¹ Department of Physics, Beijing Normal University, Beijing 100875, China

² Key Laboratory of Beam Technology and Material Modification of Ministry of Education, College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China

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We present a one-step deterministic entanglement purification protocol with linear optics and postselection. Compared with the Simon-Pan protocol (C. Simon and J. W. Pan, Phys. Rev. Lett. **89**, 257901 (2002)), this one-step protocol has some advantages. First, it can obtain a maximally entangled pair with only one step, instead of improving the fidelity of less-entangled photon pairs by performing the protocol repeatedly in other protocols. Second, it works in a deterministic way, not a probabilistic one, which greatly reduces the number of entanglement resources needed. Third, it does not require the polarization state be entangled; only spatial entanglement is needed. Moreover, it is feasible with current techniques (Nature **423**, 417 (2003)). All these advantages make this one-step protocol more convenient than others in quantum-communication applications.

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Entanglement is of vital importance for quantum communication and computation, especially the distribution of entanglement between distant locations [1–7]. However, entangled quantum systems inevitably suffer from channel noise when the entangled photons propagate away from each other. The channel noise can be caused by thermal fluctuation, vibration, the imperfection of the fiber, and so on. Thus, a maximally entangled photon pair may become a less entangled pair. That is, it is in a mixed state. Entanglement purification protocols [8–12] are essentially used to get some maximally entangled photon pairs from a set of less-entangled photon pairs with the help of local operations and classical communications. Now, it has become one of the most important stages in quantum repeaters protocol [13] and long-distance communication protocols [14, 15].

The first entanglement purification protocol (EPP) was proposed by Bennett *et al.* in 1996 [8]. Their protocol is based on controlled-NOT (CNOT) gates and is used to purify a special state, called a Werner state [16]. In the same year, Deutsch *et al.* optimized the first EPP with two additional specific unitary operations [9]. With the same logical operation, Murao *et al.* has proposed an EPP for purifying multipartite quantum systems in a Greenberger-Horne-Zeilinger state [17]. Purification of high-dimension qubit protocols also have been proposed with similar quantum logical operations [18, 19]. However, it is very difficult to implement a perfect CNOT gate with linear optics in current experiments. In 2001, Pan *et al.* proposed an EPP with linear optical elements and an ideal entanglement source [10], calling it the polarizing beam splitter (PBS) protocol. The PBS protocol was demonstrated in 2003 [20]. In 2002, Simon

and Pan proposed another polarization EPP using spatial entanglement [11], calling it the Simon-Pan protocol. In this EPP, they use the currently available parametric down-conversion (PDC) source to substitute the ideal single-pair entanglement source. In 2008, an EPP using cross-Kerr nonlinearity [12] was proposed and it is used to purify the entangled photon pairs from a PDC source. In this protocol, the cross-Kerr nonlinearity is used to construct a quantum nondemolition detector (QND) for parity-check measurements. It can be repeated to get a high-fidelity entangled photon pairs from a practical entanglement source.

Although there are some important EPPs [8–12], none of the conventional entanglement purification protocols (CEPPs) can, however, actually obtain a maximally entangled photon pairs perfectly. They can only increase the fidelity of the entangled photon pairs in an ensemble in a mixed state by consuming largely less-entangled photon pairs. For instance, after each purification step, the new fidelity F' and the initial fidelity F satisfy the following relation [8],

$$F' = \frac{F^2 + \frac{1}{9}(1-F)^2}{F^2 + \frac{2}{3}F(1-F) + \frac{5}{9}(1-F)^2}. \quad (1)$$

One can easily find that F' increases when $F > \frac{1}{2}$, but $F = 1$ is the local attractor and it cannot actually be reached. Recently, a deterministic polarization entanglement purification protocol (DEPP) has been proposed with hyperentanglement [21]. The task of deterministic entanglement purification can be accomplished with two steps: one for correcting the bit-flip errors and the other for the phase-flip errors. The two parties in quantum communication, say Alice and Bob, can correct the bit-flip errors completely in principle with the entanglement in the spatial degree of freedom and they can correct the phase-flip errors perfectly in principle with the entanglement in the frequency degree of freedom. This two-step

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†Corresponding author: fgdeng@bnu.edu.cn

DEPP does not require that the photon pairs have entanglement in the polarization degree of freedom. Different from CEPPs, in which Alice and Bob can only improve the fidelity of the remaining ensemble, they can obtain a maximally entangled photon pair from each pair in hyperentanglement with this two-step DEPP [21]. However, the entanglement in two degrees of freedom of photons increases the difficulty of its implementation.

In fact, Simon and Pan [11] used spatial entanglement to purify the polarization entanglement in 2002. In the Simon-Pan protocol [11], the two photons in each pair are entangled in two degrees of freedom of photons; that is, they are hyperentangled in the spatial and polarization degrees of freedom. However, it is only a CEPP. With a new decomposition method, the Simon-Pan protocol can be improved to be a DEPP. In this Brief Report, we present a one-step DEPP with simple linear optics and a practical PDC source by improving the Simon-Pan protocol. Alice and Bob can obtain the maximally entangled pairs without largely consuming the less-entangled pairs but rather only the spatial entanglements by postselection. Compared with the Simon-Pan protocol [11], this one-step DEPP has some advantages. First, it works in a deterministic way, not a probabilistic one. That is, Alice and Bob can obtain a maximally entangled state from each entangled photon pair, which reduces a great deal of entanglement resources consumed in quantum communication. Second, it does not require hyperentanglement in more than two degrees of freedom, nor does it require that the polarization state be entangled. Moreover, it is feasible with current techniques as its implementation is not more difficult than the experimental demonstration shown in Ref. [20]. All these advantages make this one-step protocol more convenient than other EPPs in the applications in quantum communication.

Before we start the present DEPP, let us first explain the generation of the spatial entanglement and polarization entanglement with a PDC source. As shown in Fig.1, the pump pulse of ultraviolet light passes through a beta barium borate crystal. Then a correlated pair of photons will be produced with probability p in modes a_1 and b_1 . The pulse can also be reflected and traverses the crystal a second time, producing another correlated pair into modes a_2 and b_2 with the same probability p . One also gets two pairs from one source or one pair from each source with the same order of magnitude p^2 . As $p \ll 1$, one can omit the probability p^2 because it is a second-order term. Then we can describe approximately the PDC sources with the Hamiltonian [11, 12]

$$H = \gamma[(a_{1H}^\dagger b_{1H}^\dagger + a_{1V}^\dagger b_{1V}^\dagger) + re^{i\theta}(a_{2H}^\dagger b_{2H}^\dagger + a_{2V}^\dagger b_{2V}^\dagger)] + H.c. \quad (2)$$

where subscripts H and V denote horizontal and vertical polarizations, respectively. r is the relative probability of emission of photons into the lower modes ($a_2 b_2$) compared to the upper modes ($a_1 b_1$), and θ is the phase between these two possibilities. We can make $r = 1$ and $\theta = 0$ as a simple case, the same as in the

Simon-Pan protocol [11]. One can see that the item $(a_{1H}^\dagger b_{1H}^\dagger + a_{1V}^\dagger b_{1V}^\dagger + a_{2H}^\dagger b_{2H}^\dagger + a_{2V}^\dagger b_{2V}^\dagger)|0\rangle$ represents the entanglement in both the polarization and the spatial modes. So this single-pair state can also be written as

$$|\Psi\rangle = \frac{1}{2}(|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)(|H\rangle|H\rangle + |V\rangle|V\rangle)_{ab} \quad (3)$$

in a different notation. The subscripts a and b represent the two photons a and b sent to Alice and Bob, respectively. If we denote $|\phi^+\rangle_p = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)_{ab}$ and $|\phi^-\rangle_s = \frac{1}{\sqrt{2}}(|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$, Eq.(3) can also be rewritten as:

$$\rho = \rho_p \otimes \rho_s, \quad (4)$$

where $\rho_p = |\phi^+\rangle_p \langle\phi^+|$ and $\rho_s = |\phi^-\rangle_s \langle\phi^-|$.

Now we start our purification protocol. Its principle is shown in Fig.1. If the two photons do not suffer from decoherence and remain in the state with the form of Eq.(3), the whole state evolves as:

$$\begin{aligned} |\Psi\rangle &= \frac{1}{2}(|H_{a1}\rangle|H_{b1}\rangle + |V_{a1}\rangle|V_{b1}\rangle + |H_{a2}\rangle|H_{b2}\rangle + |V_{a2}\rangle|V_{b2}\rangle) \\ &\rightarrow \frac{1}{2}(|H_{c1}\rangle|H_{d1}\rangle + |V_{e1}\rangle|V_{f1}\rangle + |V_{c2}\rangle|V_{d2}\rangle + |H_{e2}\rangle|H_{f2}\rangle). \end{aligned} \quad (5)$$

One can see that the items $|H_{c1}\rangle|H_{d1}\rangle$ and $|V_{c2}\rangle|V_{d2}\rangle$ will be detected in D_2 and D_4 , and $|V_{e1}\rangle|V_{f1}\rangle$ and $|H_{e2}\rangle|H_{f2}\rangle$ will be detected in D_5 and D_7 . That is, if the coincidence detectors D_2 and D_4 , or D_5 and D_7 click (i.e., the two photons emit from the outputs D_2 and D_4 or D_5 and D_7), Alice and Bob can get the maximally entangled state $|\phi^+\rangle_p = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$ by postselection.

So far we have been talking about the absence of noise in an ideal case. In a practical transmission, the channel noise always exists and makes the pure state become a mixed one. The polarization part of the entangled state may suffer from bit-flip and phase-flip errors, and the phase between the modes $a_1 b_1$ and $a_2 b_2$ is not exactly stable; that is, the spatial entanglement will also be affected. Fortunately, by having them in two different optical fibers, the bit-flip errors of spatial entanglement (i.e., the two spatial modes on each side) are extremely small. Here we suppose that only the polarization entanglement part suffers from noise, just as in the Simon-Pan protocol [11]. Through a noisy channel, Eq.(4) becomes:

$$\rho' = \rho'_p \otimes \rho_s, \quad (6)$$

where

$$\begin{aligned} \rho'_p &= F|\phi^+\rangle_p \langle\phi^+| + F_1|\phi^-\rangle_p \langle\phi^-| \\ &\quad + F_2|\psi^+\rangle_p \langle\psi^+| + F_3|\psi^-\rangle_p \langle\psi^-|. \end{aligned} \quad (7)$$

Here $F + F_1 + F_2 + F_3 = 1$, $|\phi^-\rangle_p = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle - |V\rangle|V\rangle)_{ab}$, and $|\psi^\pm\rangle_p = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle \pm |V\rangle|H\rangle)_{ab}$. Based on the density matrix of Eq.(7), we denote that there are

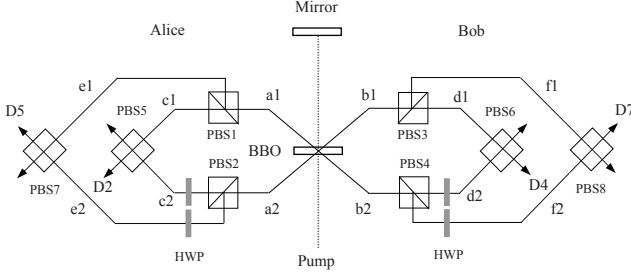


FIG. 1: Schematic illustration of the one-step deterministic entanglement purification protocol. The two parametric down-conversion sources are used to generate the photons being entangled both in the polarization and in the spatial-mode degrees of freedom of photons. In the postselection after the polarizing beam splitters (PBSs), the photon pairs have been purified in a deterministic way. The PBS is used to transmit the $|H\rangle$ polarization photons and reflect the $|V\rangle$ polarization photons. The half-wave plates (HWPs) can convert the polarization state $|H\rangle$ into $|V\rangle$, or vice versa. Only four detectors are used here. If the measurement results are D_2 and D_4 , or D_5 and D_7 , the two photons are in the state $|\phi^+\rangle$. If the measurement results are D_2 and D_7 or D_4 and D_5 , the two photons are in the state $|\psi^+\rangle$.

two kinds of errors in the mixed state. One is the bit-flip error and the other is the phase-flip error. For instance, if the state $|\phi^+\rangle_p$ becomes $|\psi^+\rangle_p$, we call it a bit-flip error. If $|\phi^+\rangle_p$ becomes $|\phi^-\rangle_p$, a phase-flip error occurs. $|\psi^-\rangle_p$ contains both a bit-flip error and a phase-flip error. So the whole task of entanglement purification is to correct these two kinds of errors.

If a bit-flip error takes place with a probability of F_2 , after the PBSs and half-wave plates (HWPs) shown in Fig.1, the state becomes:

$$\begin{aligned} & \frac{1}{2}(|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)(|H\rangle|V\rangle + |V\rangle|H\rangle)_{ab} \\ &= (|H_{a1}\rangle|V_{b1}\rangle + |H_{a2}\rangle|V_{b2}\rangle + |V_{a1}\rangle|H_{b1}\rangle + |V_{a2}\rangle|H_{b2}\rangle) \\ &\rightarrow \frac{1}{2}(|H_{c1}\rangle|V_{f1}\rangle + |V_{c2}\rangle|H_{f2}\rangle + |V_{e1}\rangle|H_{d1}\rangle + |H_{e2}\rangle|V_{d2}\rangle). \end{aligned} \quad (8)$$

We can also find that the items $|H_{c1}\rangle|V_{f1}\rangle$ and $|V_{c2}\rangle|H_{f2}\rangle$ have the same outputs D_2 and D_7 , and the items $|V_{e1}\rangle|H_{d1}\rangle$ and $|H_{e2}\rangle|V_{d2}\rangle$ have the same outputs D_4 and D_5 . That is, if the coincidence detectors D_2 and D_7 or D_4 and D_5 click, Alice and Bob can finally obtain the maximally entangled state $\frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle)$ by postselection. Therefore, by performing a bit-flip operation $\sigma_x = |H\rangle\langle V| + |V\rangle\langle H|$ on one of the two photons, Alice and Bob can get rid of the bit-flip error and then obtain the uncorrupted state $|\phi^+\rangle_p = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$.

Following the same principle discussed above, if the initial state is $\frac{1}{2}(|H\rangle|H\rangle - |V\rangle|V\rangle)_{ab}(|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$, in other words, if a phase-flip occurs with a probability of F_1 , after the PBSs and the HWPs, the coincidence detectors will get the results that D_2 and D_4 , or D_5 and D_7 click. This result is the same as the

case with no phase-flip error [see Eq.(4)]. If both a bit-flip error and a phase-flip error occur, the initial state is $\frac{1}{2}(|H\rangle|V\rangle - |V\rangle|H\rangle)_{ab}(|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$, which has the same result as the case with only bit-flip errors [see Eq.(8)]. Surprisingly, the phase-flip error has no effect on the entanglement purification here, which is far different from the CEPPs [8–12].

So far we have described the principle of our one-step DEPP. With one-step DEPP, Alice and Bob can obtain the maximally entangled state $\frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$ or $\frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle)$, corresponding to the coincidence detectors and two deterministic output modes. That is, Alice and Bob can implement deterministic entanglement purification by postselection. Compared with the initial mixed state in Eq.(6), the spatial entanglement has been consumed but polarization entanglement remains.

The present one-step protocol for deterministic entanglement purification is interesting because of its great simplicity and high efficiency. In CEPPs [8–10], Alice and Bob should use two less-entangled copies to complete a round for purification. After some local operations and classical communication, one photon pair in a high-fidelity entangled state will remain with some probability. In this case, at last one of the two less-entangled pairs is consumed. If we want to obtain higher-quality entangled pairs, more less-entangled pairs are consumed. Surprisingly, in our one-step DEPP, the entanglement in the polarization seems redundant and it does not require the polarization part of the state be entangled; that is, the coefficients of F , F_1 , F_2 , and F_3 only need to satisfy the condition $F + F_1 + F_2 + F_3 = 1$. But in the CEPPs [8–12], the initial fidelity F should satisfy the condition $F > \frac{1}{2}$ to ensure the mixed state is entangled in the polarization degree of freedom. Because local operations and classical communication can not increase entanglement, entanglement purification is essentially the transformation of the entanglement. In the conventional protocols [8–10], its transformation is completed between the same kind of entanglement (i.e., polarization entanglement) but with different particles. The Simon-Pan protocol [11] and the present one-step DEPP accomplish the purification with the transformation between the polarization entanglement and the spatial entanglement. However, the spatial entanglement in the Simon-Pan protocol [11] is only used to purify the bit-flip error. After consuming the entanglement in the spatial degree of freedom, Alice and Bob have to adopt the CEPPs [10, 11] to improve the fidelity of the polarization entanglement. However, in the QND protocol [12], the QND has the function of both a nondestructive single-photon detector and a CNOT gate, and it also cannot reach the maximally entangled pure state, and more less-entangled pairs will be consumed if the QND protocol is performed repeatedly.

To understand why this one-step DEPP has a success probability of 100% for entanglement purification in principle, we employ another example to explain this method. From the view of the outcome of the measurement on the photon systems in the polarization degree of freedom

with a product basis, say $\sigma_z^A \otimes \sigma_z^B$, the state of the polarization part after the transmission over noisy channels can be described as:

$$\rho_p'' = \alpha |HH\rangle\langle HH| + \beta |VV\rangle\langle VV| + \gamma |HV\rangle\langle HV| + \delta |VH\rangle\langle VH|, \quad (9)$$

where α, β, γ , and δ represent the probabilities that Alice and Bob obtain the product states $|HH\rangle, |HV\rangle, |VH\rangle$, and $|VV\rangle$, respectively, when they measure their photon pair with the basis $\sigma_z^A \otimes \sigma_z^B$. $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$. That is, ρ' can be rewritten as:

$$\rho' = \rho_p'' \otimes \rho_s. \quad (10)$$

So ρ' can be viewed as a probabilistic mixture of four pure states; this pair is in the state $|HH\rangle \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$, $|VV\rangle \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$, $|HV\rangle \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$, or $|VH\rangle \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$ with the probability of α, β, γ , or δ , respectively.

Following the same principle above, $|HH\rangle \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$ leads to the two-mode case in the output D_2 and D_4 , and becomes the maximally entangled state $\frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$. $|VV\rangle \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$ has the same result above but leads to the output D_5 and D_7 . $|HV\rangle \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$ is in the output D_2 and D_7 with the remaining state $\frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle)$, and $|VH\rangle \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$ will have the same result but leads to the output D_4 and D_5 . We know that in each item, for instance, the state $|HH\rangle \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$, there is no entanglement in polarization, but the spatial part is entangled. After purification, the spatial entanglement has been transformed into the polarization degree of freedom successfully.

It is interesting to compare this protocol with the Simon-Pan protocol [11]. In their protocol, the spatial entanglement is used to purify polarization entanglement, by selecting both upper and lower modes, and then Alice and Bob can correct the bit-flip error successfully. However, the phase-flip error can not be purified directly. They have to use the CEPPs [8, 9] to get rid of it. It is well known that a mixed state can be decomposed in any orthogonal basis. For example, like Eq.(6), it is decomposed in a Bell-state basis, but in Eq.(10) it is in a product-state basis. In Eq.(6), there are two kinds of errors, bit-flip error and phase-flip error, but no phase-flip error exists in Eq.(10). The state may be one bit-flip error, like $|HV\rangle$ or $|VH\rangle$, or two bit-flip errors, like $|VV\rangle$. In this one-step DEPP, the initial mixed state has been

first divided into different bit-flip errors ($|HH\rangle, |VV\rangle, |HV\rangle$, or $|VH\rangle$) and guided into different spatial modes with PBS_1, PBS_2, PBS_3 , and PBS_4 . Then another four PBSs are used to convert the spatial entanglement into the polarization entanglement perfectly by postselection. This is the reason that $|\phi^+\rangle_p \otimes |\phi\rangle_s$ and $|\phi^-\rangle_p \otimes |\phi\rangle_s$ have the same outputs and the phase-flip error has been eliminated automatically. However, in the Simon-Pan protocol [11], the two PBSs can only be used to discriminate the bit-flip error and the phase-flip error remains. So the transformation efficiency is only 50%. Moreover, after the single-photon detector, the fidelity can not be increased any more because the spatial entanglement is consumed completely.

In summary, we have presented a one-step deterministic entanglement purification protocol with simple linear optical elements. Compared with other protocols, this protocol has several advantages. First, it can obtain a maximally entangled pair with only one step, instead of improving the fidelity of less-entangled pairs by performing the purification protocol repeatedly. Second, Alice and Bob do not need many less-entangled pairs because this one-step protocol works in a deterministic way, not a probabilistic one. This one-step protocol can greatly reduce the number of entanglement resources needed. Third, it does not require that the polarization state be entangled; only spatial entanglement is needed. The most important advantage of this one-step DEPP may be its realization with current technology. In a previous experiment [20], the main experimental requirements of good mode overlap on the PBSs and phase stability in the spatial mode have both been achieved. All these advantages make this one-step protocol more convenient than others [8–12, 21] in current quantum communication.

Although we only discuss the present one-step DEPP with spatial entanglement, it works with frequency entanglement if the latter suffers less from the channel noise. That is, the entanglement transformation can also be accomplished between the frequency degree of freedom and the polarization degree of freedom, and the method for the decomposition of quantum states in this one-step DEPP is general for other degrees of freedom of photons.

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